

Normality Test for Financial Variables

Test di normalità per variabili finanziarie

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Riassunto: I test di normalità generalmente usati per le variabili finanziarie non consentono sempre di valutare correttamente se la distribuzione normale rappresenta un riferimento efficace per la misura del rischio. Da un lato, infatti, la curtosi può non essere causata dalla coda sinistra della distribuzione, dall'altro, i piccoli campioni frequentemente adoperati non consentono la validità delle proprietà asintotiche dei test. Mediante il ricorso a un esperimento Montecarlo, viene proposta una procedura non parametrica, particolarmente utile nel caso di piccoli campioni, per valutare se l'adozione dell'ipotesi di normalità conduca ad una sottostima del rischio finanziario.

Keywords: Normality test, financial variables, risk measurement.

1. Introduction

Normal distribution represents the key assumption of the traditional (mean-variance) portfolio theory by Markowitz (1952) and of the usual measures of risk. Since divergences from the normal distribution are widely documented for financial variables, the immediate consequence is particularly harmful, because it leads to a misleading evaluation of the risk, and, especially, to underestimate it.

Divergence from normality is usually measured by means of normality tests, like Jarque-Bera and Lilliefors test, where a fundamental role is played by the kurtosis. Since financial return distributions are typically leptokurtic, these tests often refuse the hypothesis of normality.

However, it is important to stress how kurtosis could be determined by three distinct conditions, which are not necessarily simultaneous: extreme negative frequencies, central frequencies and extreme positive frequencies higher than under normality.

From a risk management perspective only the first one is relevant, while the others even induce less risky conditions. Nevertheless, traditional normality tests do not allow to distinguish among the causes of the excess of kurtosis and therefore can lead to an increase in the error of the I type, i.e. to reject the normality assumption when it could be acceptable in order to measure the risk.

Moreover, risk measurement is often performed on a small amount of observations: long past data are seldom useful, and for some frequencies, such as monthly or annual data, also a long period contains few observations. This feature implies that the methodologies used should embed the required small sample adjustment.

By resorting to a Montecarlo study, the paper introduces a non parametric method able to correctly evaluate if normal distribution represents a reliable reference for financial risk measurement in small samples.

2. Methodology

One of the most widespread and frequently used normality test for financial data is the Bowman-Shenton test (Bowman and Shenton, 1975), usually known as the Jarque–Bera test (Jarque and Bera, 1987), that is a two-sided test based on the sample skewness g_1 and kurtosis g_2 . Under the null hypothesis of normality, the test statistic JB has an asymptotic χ^2 distribution with 2 degrees of freedom.

In the non parametric setting, an important reference for normality testing has been proposed by Lilliefors (1967) with a relevant variant of Kolmogorov test. In Lilliefors test, data have to be standardized, $z = (x_j - \bar{x}) / s$, and the test statistic is expressed as

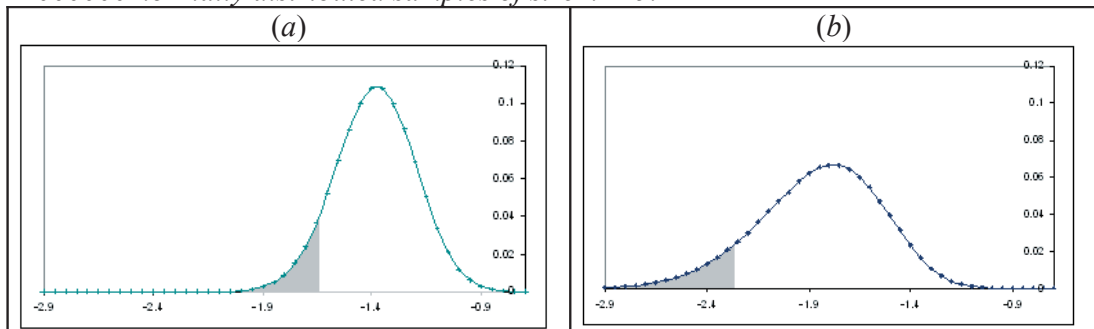
$$D_n = \max_{j=1}^n |F_n(z_j) - \Phi_n(z_j)|$$

where $\bar{x}$ is the sample mean, s the unbiased estimate of the sample standard deviation, $F_n(z_j)$ is the empirical cumulative function and $\Phi_n(z_j)$ is the standard normal cumulative function. Normality assumption is rejected at the significance level of α if D_n exceeds the $(1 - \alpha)$ percentile calculated by Lilliefors.

Both Jarque-Bera and Lilliefors test are based on all sample information, while in the measurement of risk, only left tail of distribution is relevant. More specifically, departures from normality are relevant only if related to the left tail, and therefore to the likely underestimation of risk. Furthermore, financial variables are frequently analyzed on the basis of only few observations, and therefore asymptotic properties of testing procedures could be not valid.

In order to assess the behavior of the left tail of distribution, a Montecarlo study is performed on 1000000 samples of size $n=25$ randomly extracted from a standard normal distribution: Figure 1 illustrates the frequency distribution of the sample 10th percentile (a) and the sample 5th percentile (b). Obviously, by increasing n , the two distributions converge, respectively, to the standard normal values -1.2814 and -1.6415,

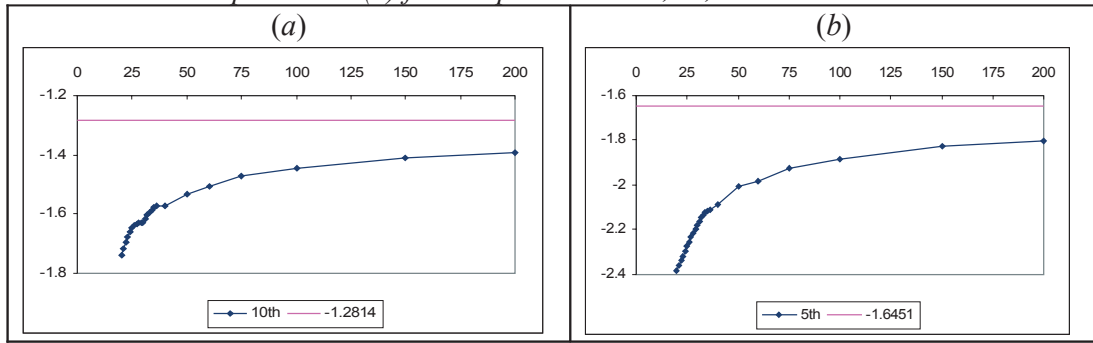
Figure 1: Sample distribution of the 10th percentile (a) and of the 5th percentile (b) in 1000000 normally distributed samples of size $n=25$.



but for small samples it is evident a strong variability, and to refer to asymptotic values -1.2814 for the 10th percentile and -1.6415 for the 5th percentile could be greatly misleading.

From the other side, it could be reasonable to make use of the information provided by the frequency distribution of sample percentile: one particular percentile of the distribution can be interpreted as a significance level in order to detect departure from normality. Figure 2 illustrates, for some values of n , the 10th percentile of the sample distribution of 10th percentile (a) and of the sample distribution of the 5th percentile (b). For small values of n the distance from the standard values is quite remarkable.

Figure 2: 10th percentile of the sample distribution of the 10th (a) and the 5th (b) standard normal percentile (a) for sample size $n=20, \dots, 200$.



In the non parametric framework of Lilliefors test, and by using information illustrated in Figures 1 and 2, it is possible to propose a new decision rule:

$$D_{v,n} = (z_v - z_{v,\alpha}^*)$$

where z_v is the v^{th} empirical percentile and $z_{v,\alpha}^*$ is the α percentile of the sample distribution of the v^{th} standard normal percentile as in Figure 1. Figure 2 shows in panel a $z_{v=10,\alpha=0.1}^*$ and in panel b $z_{v=5,\alpha=0.1}^*$. For example, in order to detect if normal distribution correctly measure the risk of a financial variable at the significance level $\alpha=0.1$, it is sufficient to compare the 10th empirical percentile $z_{v=10}$ with values $z_{v=10,\alpha=0.1}^*$ in Figure 2a: normal distribution underestimates the risk only if $D_{v=10,n} < 0$, i.e. if empirical value $z_{v=10}$ exceeds theoretical value $z_{v=10,\alpha=0.1}^*$.

3. Results and conclusions

In Table 1 are compared some results related to samples of size $n=25$ extracted from a standard normal distribution and samples of size $n=25$ derived from Dow Jones Industrial Index returns during the period January 1970 – December 2005.

In 1000000 samples extracted from a standard normal distribution, Jarque-Bera test indicates, at the significance level $\alpha=0.10$, a normal distribution in 97% of cases, but in 75.14% of samples the 10th percentile is lower than the standard value -1.2814.

In particular, 74.02% of samples are normally distributed according to Jarque Bera test, but extreme negative observations are more frequent than in the normal distribution: that is a quite unsatisfactory combination, especially if concerned about risk measurement. Finally, 1.88% of samples are classified as not normally distributed, but their extreme negative observations do not show departure from normality.

By using values derived from the sample distribution of the 10th percentile in samples of size $n=25$ extracted from a standard normal distribution results are strongly different: 87.74% of samples are both normally distributed and their 10th percentile is not lower than the limit -1.6457. The statistic $D_{v=10, n=25}$ is negative, at the significance level $\alpha = 0.10$, in 9.75% of samples normally distributed, and allows both to detect samples with left tail inconsistent with normal distribution and to fix significance level α , i.e. error probability if rejecting normality.

The results related to samples of size $n=25$ extracted from Dow Jones index returns offer again an analogous situation: traditional testing procedure does not allow to identify samples with left tail significantly different from normal case, while by resorting to $D_{v,n}$, and comparing empirical v^{th} percentile with value $z_{v,\alpha}^*$, it is possible to correctly evaluate risk profiles. Only 9.27% of cases for daily data, and 7.86% for monthly data, show a significant departure from normal distribution.

Table 1: *Left tail and normal distribution in samples of size $n=25$.*

		Normal data		DJ daily data		DJ monthly data	
		Jarque Bera test		Jarque Bera test		Jarque Bera test	
Left tail		normal	rejects	normal	rejects	normal	Rejects
-1.2814	>	22.98	1.88	25.69	7.40	22.11	13.76
	≤	74.02	1.12	64.68	2.23	62.16	1.97
-1.6457	>	87.74	2.76	81.10	9.45	76.41	15.72
	≤	9.75	0.25	9.27	0.18	7.86	0

Since, in financial variables analysis, importance of normal distribution is related to risk measurement, it should be straightforward to complete information provided by standard normality tests with an evaluation of normality based on the left tail of financial returns distribution. The non parametric statistic $D_{v,n}$ developed in the paper allows to achieve this goal in the framework of small samples, which are frequently used in empirical risk analysis.

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